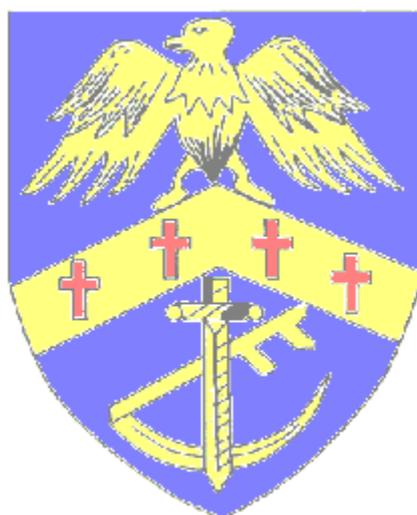


SUNNYMEDE JUNIOR SCHOOL

WRITTEN CALCULATIONS POLICY



Ratified by Governing Body: June 2015

Review: Every Two Years

This Review: June 2017

Next Review Date: June 2019

Review date	Headteacher's signature	Chair of Governor's signature
June 2017		

Rationale and Aims

This policy outlines a model progression through written strategies for addition, subtraction, multiplication and division in line with the new 2014 National Curriculum. As children move at the pace appropriate to them, teachers will be presenting strategies and equipment appropriate to children's level of understanding. However, it is expected that the majority of children in each class will be working at age-appropriate levels as set out in the National Curriculum 2014 and in line with school policy.

This policy aims to support all staff in their understanding of teaching written methods for the four rules of addition, subtraction, multiplication and division. It contains the key pencil and paper procedures that will be taught within our school. It aims to ensure consistency, continuity and progression throughout the school and reflects a whole school agreement. However, it does not set out a system of progression that every child must work through in a set order, as strategies taught to individuals and groups of children will reflect the School's commitment to a personalised curriculum, with methods differentiated accordingly. An abridged version of this policy will be distributed to parents.

Pupils will progress from pictorial/ mental methods to written methods, once their depth of understanding is secure in both fluency and reasoning situations. Once one operation is secure, pupils may need to revisit more informal methods to support their understanding of other operations of mathematical concepts.

While this policy focuses on written calculations in mathematics, we recognise the importance of the mental strategies and known facts that form the basis of all calculations. The following checklists outline the key skills and number facts that children are expected to develop throughout the school.

To add and subtract successfully, children should be able to:

- recall all addition pairs to $9 + 9$ and number bonds to 10
- recognise addition and subtraction as inverse operations
- add mentally a series of one digit numbers (e.g. $5 + 8 + 4$)
- add and subtract multiples of 10 or 100 using the related addition fact and their knowledge of place value (e.g. $600 + 700$, $160 - 70$)
- partition 2 and 3 digit numbers into multiples of 100, 10 and 1 in different ways (e.g. partition 74 into $70 + 4$ or $60 + 14$)
- use estimation by rounding to check answers are reasonable

To multiply and divide successfully, children should be able to:

- add and subtract accurately and efficiently
- recall multiplication facts to $12 \times 12 = 144$ and division facts to $144 \div 12 = 12$
- use multiplication and division facts to estimate how many times one number divides into another etc.
- know the outcome of multiplying by 0 and by 1 and of dividing by 1
- understand the effect of multiplying and dividing whole numbers by 10, 100 and later 1000
- recognise factor pairs of numbers (e.g. that $15 = 3 \times 5$, or that $40 = 10 \times 4$) and increasingly able to recognise common factors
- derive other results from multiplication and division facts and multiplication and division by 10 or 100 (and later 1000)

- notice and recall with increasing fluency inverse facts
- partition numbers into 100s, 10s and 1s or multiple groupings
- understand how the principles of commutative, associative and distributive laws apply or do not apply to multiplication and division
- understand the effects of scaling by whole numbers and decimal numbers or fractions
- understand correspondence where n objects are related to m objects
- investigate and learn rules for divisibility

In every written method there is an element of mental processing. Sharing written methods with the teacher encourages children to think about the mental strategies that underpin them and to develop new ideas. Therefore written recording both helps children to clarify their thinking and supports and extends the development of more fluent and sophisticated mental strategies. Children will be encouraged to see mathematics as both a written and spoken language. Teachers will support and guide children through the following important stages:

- Developing the use of pictures and a mixture of words and symbols to represent numerical activities;
- Using standard symbols and conventions;
- Use of jottings to aid a mental strategy;
- Use of pencil and paper procedures;
- Use of a calculator as a checking device.

This policy concentrates on the introduction of standard symbols, the use of the empty number line as a jotting to aid mental calculation and on the introduction of pencil and paper procedures. It is important that children do not abandon jottings and mental methods once pencil and paper procedures are introduced. Therefore, children will always be encouraged to look at a calculation/problem and then decide the best method to choose – pictures, mental calculation with or without jottings, structured recording or a calculator. Our long-term aim is for children to be able to select an efficient method of their choice that is appropriate for a given task.

It is expected that:

- *mental calculation methods will be practised regularly alongside their learning of written calculation methods*
- addition/subtraction and multiplication/division methods will be taught alongside each other so that pupils can see and use the relationship between them
- pupils will be taught to recognise and use a range of vocabulary relating to each of the four rules.
- models and images will be used to support learning of methods at all ages and stages, if required, though particularly for pupils with special education needs
- pupils will be taught to estimate their answers first and to check calculations with a variety of strategies including the inverse operation.
- pupils will have opportunities to use and apply mental and written strategies in a range of problem solving, investigative and 'real-life' contexts

WRITTEN METHODS OF ADDITION

Upon entry to Year 3, children will use a variety of informal jottings

Method 1

$$32 + 25$$

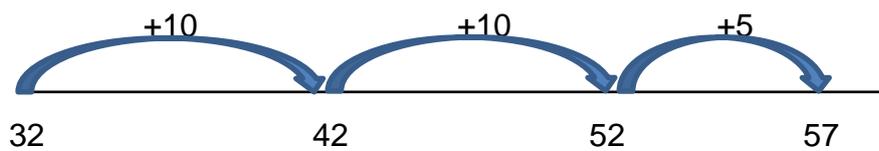
50
7

$$50 + 7 = 57$$

Method 2

$$32 + 25$$
$$30 + 2 + 20 + 5 = 57$$

Method 3



Teaching point:

The Year 3 teachers will ascertain which method(s) the children in their group have been taught in KS1. Using this knowledge, they will support and scaffold progression onto more formal methods of recording (Stage 1, see below) using the method(s) that the children currently use.

Stage 1 – Partitioning using 2 digit numbers

Without carrying (e.g. 32+25):

$$\begin{array}{r} \text{T} \quad 30 + 20 = 50 \\ \text{O} \quad 2 + 5 = 7 \\ \hline 50 + 7 = 57 \\ \hline \end{array}$$

With carrying (e.g. 35+27):

$$\begin{array}{r} \text{T} \quad 30 + 20 = 50 \\ \text{O} \quad 5 + 7 = 12 \\ \hline 50 + 12 = 62 \\ \hline \end{array}$$

Stage 2 – Expanded column addition using 2 digit numbers

Without carrying (e.g. 32+25):

$$\begin{array}{r} \text{T} \quad \text{O} \\ 30 + 2 \\ + 20 + 5 \\ \hline 50 + 7 \\ \hline \end{array}$$

With carrying (e.g. 35+27):

$$\begin{array}{r} \text{T} \quad \text{O} \\ 30 + 5 \\ + 20 + 7 \\ \hline 50 + 12 \\ \hline \end{array}$$

Moving on to:

$$\begin{array}{r} \text{T} \quad \text{O} \\ 30 + 5 \\ + 20 + 7 \\ \hline 60 + 2 \\ \hline \cancel{10} \end{array}$$

Teaching points:

Children should be encouraged to lay their calculations out in columns and to label the columns
Children should always add the digits in the 'ones' column first
Children should strike through the number that has been carried once they have added it to the column total
Extend to three digit numbers

Stage 3 – Compact column method (Children will be expected to use this method to add whole numbers together by the end of Year 4)

First step (with column labels):

$$\begin{array}{r} \text{T} \quad \text{O} \\ 3 \quad 5 \\ + 2 \quad 7 \\ \hline 6 \quad 2 \\ \hline \cancel{1} \end{array}$$

Second step (without column labels):

$$\begin{array}{r} 35 \\ + 27 \\ \hline 62 \\ \hline \cancel{1} \end{array}$$

Teaching points:

Extend to three digit numbers. This concept will be taught in gradual stages. In the first instance, the calculations will require carrying from the 'ones' to the 'tens' column, then the 'tens' to the 'hundreds' column. Finally, carrying will be required from the 'ones' to the 'tens' and the 'tens' to the 'hundreds' within the same calculation

Extend to decimal numbers.

Progression within Addition:

Year 3 – Objective(s): Add numbers up to 3 digits; Use partitioning method for addition to add two or three 3-digit numbers or three 2-digit numbers; Begin to use compact column addition to add numbers with three digits.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, units, ones, tens, hundreds, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', expanded, compact, digits, inverse

Key Skills for addition at Year 3

- Know pairs with each total to 20
- Know pairs of multiples of 10 with a total of 100
- Add any two 2-digit numbers by counting on in 10s and 1s or by using partitioning
- Add multiples and near multiples of 10 and 100
- Add 1, 10, 100 to 3-digit numbers
- Understand place value in 3-digit numbers
- Perform place value additions without a struggle. (E.g. $300 + 8 + 50 = 358$)
- Use place value and number facts to add a 1-digit or 2-digit number to a 3-digit number (E.g. $104 + 56$ is 160 since $104+50=154$ and $6+4=10$ and $676 + 8$ is 684 since $8=4+4$ and $76+4+4=84$)
- Add pairs of 3-digit numbers mentally, which do not bridge the tens or hundreds barrier, e.g. $320 + 450$
- Begin to add amounts of money using partitioning.
- Solve problems with addition using number facts, place value, missing numbers.

Year 4 – Objective(s): Add numbers with up to 4 digits

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, units, ones, tens, hundreds, thousands, partition, addition, column, tens boundary, hundreds boundary, thousands boundary, increase, vertical, 'carry', expanded, compact, digits, inverse

Key Skills for addition at Year 4

- Select appropriate method, mental, jottings, written—and explain why
- Use formal written methods to add 3 and 4 digit numbers
- Add any two 2-digit numbers by partitioning or counting on
- Know by heart/quickly derive number bonds to 100 (e.g. $32 + 68$) and to £1 ($64p + 36p$)
- Add to the next hundred, pound and whole number. (e.g. $234 + 66 = 300$, $3.4 + 0.6 = 4$)
- Perform place value additions without a struggle. (e.g. $300 + 8 + 50 + 4000 = 4358$)
- Add multiples and near multiples of 10, 100 and 1000.
- Add £1, 10p, 1p to amounts of money
- Use place value and number facts to add 1-, 2-, 3-and 4-digit numbers where a mental calculation is appropriate'. (e.g. $4004 + 156$ by knowing that $6+4=10$ and that $4004+150=4154$ so total is 4160)
- Perform inverse operations to check calculations

- Round any number to the nearest 10, 100 and 1000
- Solve 2-step problems in context
- Continue to practise a wide range of mental addition strategies e.g. Round and adjust, near doubles, numbers bonds, partitioning and recombining

Year 5 – Objective(s): Add numbers with more than 4 digits including money, measure and decimals with different numbers of decimal places

Key Vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, ones, units, tens, hundreds, thousands, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', expanded, compact, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths.

Key Skills for addition at Year 5:

- Locate 5 and 6 digit numbers on a marked line; use this to compare/order numbers.
- Use rounding to check accuracy
- Understand a one-place decimal number as a number of tenths and a two-place decimal number as a number of hundredths.
- Add 0.1 or 0.01 to any decimal number with confidence, e.g. $5.83 + 0.01$
- Add mentally with confidence – where the numbers are less than 100 or the calculation relies upon simple addition and place value.
- Confidently add numbers with more than 4-digits using a secure formal written method, including adding 'piles' of numbers
- Use inverse to check calculations

Year 6 – Objective(s): Add several numbers of increasing complexity including money, measure and decimals with different numbers of decimal places; Use compact column method to add in context of money, measures, including decimals with different numbers of decimal places.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, units, ones, tens, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', expanded, compact, thousands, hundreds, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths.

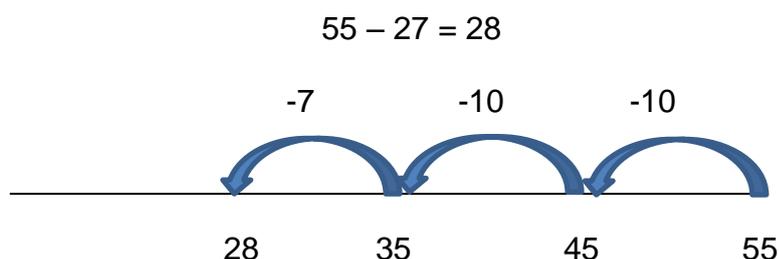
Key Skills for addition at Year 6:

- Add mentally with confidence using larger numbers and calculations of increasing complexity
- Add several large numbers using formal methods of written addition
- Add several large or decimal numbers using formal methods of written addition
- Perform mental calculations, including with mixed operations and large numbers, using a range of strategies
- Use negative numbers in context and calculate intervals across zero
- Solve multi-step problems
- Use estimation and inverse to check the validity of an answer

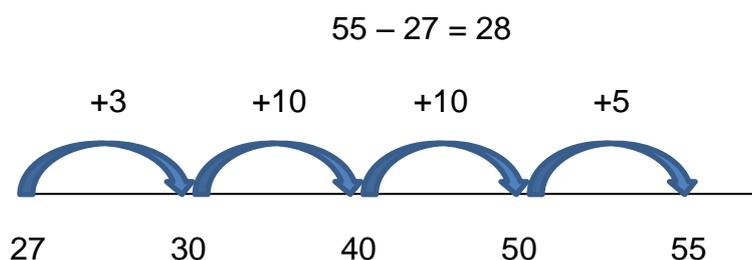
WRITTEN METHODS OF SUBTRACTION

Stage 1 - Subtraction using an empty number line

Method 1 (counting backwards):



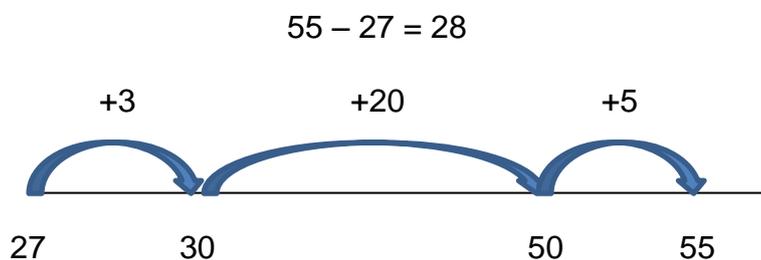
Method 2 (counting forwards):



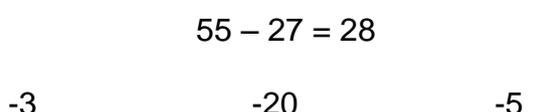
Teaching points:

Using method 2, children should be encouraged to firstly jump to the next multiple of 10. Confusion may occur as the children are counting on from the smallest to the largest number.

Method 3 (counting forward using a more efficient method):



Method 4 (counting backwards using a more efficient method):



27

30

50

55

Teaching points:

An empty number line can also be used at any stage of development for the following areas:

- Decimals e.g. 4.2 – 3.68
- Elapse time e.g. Find the difference between 3.55pm and 6.20pm
- Numbers containing zeros e.g. 6003 – 1987
- Conversion in measure (capacity/length etc.) e.g. 5 litres – 860ml

Stage 2 – Expanded column subtraction using 2 digit numbers

Without exchanging (e.g. 65-22):

$$\begin{array}{r}
 \text{T} \quad \text{O} \\
 60 \quad 5 \\
 - 20 \quad 2 \\
 \hline
 40 \quad + \quad 3 \\
 \hline
 \end{array}$$

With exchanging (e.g. 61-27):

$$\begin{array}{r}
 \text{T} \quad \text{O} \\
 50 \quad \cancel{60} \quad 10+1 \\
 - 20 \quad 7 \\
 \hline
 30 \quad + \quad 4 \\
 \hline
 \end{array}$$

Moving on to:

$$\begin{array}{r}
 \text{T} \quad \text{O} \\
 50 \quad \cancel{60} \quad 11 \\
 - 20 \quad 7 \\
 \hline
 30 \quad + \quad 4 \\
 \hline
 \end{array}$$

Teaching points:

Children should be encouraged to lay their calculations out in columns and to label the columns
 Children should always subtract the digits in the 'ones' column first
 Extend to 3 digit numbers

Stage 3 – Compact column method (decomposition) (Children will be expected to use this method to subtract whole numbers together by the end of Year 4)

Step 1 (2 digit numbers without exchanging):

$$\begin{array}{r}
 \text{T} \quad \text{O} \\
 87 \\
 - 53 \\
 \hline
 34 \\
 \hline
 \end{array}$$

Step 2 (3 digit numbers without exchanging):

$$\begin{array}{r}
 257 \\
 - 123 \\
 \hline
 134 \\
 \hline
 \end{array}$$

Step 3 (2 digit numbers with exchanging):

$$\begin{array}{r}
 \text{T} \quad \text{O} \\
 8 \quad 1 \\
 \cancel{8} \quad 1 \\
 - 3 \quad 6 \\
 \hline
 \end{array}$$

Step 4 (3 digit numbers with exchanging):

$$\begin{array}{r}
 1 \quad 12 \quad 1 \\
 \cancel{2} \quad \cancel{3} \quad 5 \\
 - 1 \quad 7 \quad 6 \\
 \hline
 \end{array}$$

Progression within Subtraction:**Year 3 – Objective(s): Subtract with 2 and 3-digit numbers**

Key vocabulary: equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least, count back, how many left, how much less is..., difference, count on, strategy, partition, tens units, exchange, digit, value, hundreds

Key Skills for subtraction at Year 3:

- Understand place value in 3-digit numbers; subtract 1s, 10s or 100s without difficulty; use this to subtract multiples of 1, 10, 100 to/from 3-digit numbers.
- Mentally subtract any pair of 2 digit numbers, e.g. 75 – 58
- Recognise that there are two ways of completing subtractions, either by counting up (using empty number line) or by counting back, e.g. 54 – 3 (counting up)
- Subtract mentally using place value and number bonds, e.g. 347-5, 347- 40, 347-100)

Year 4 – Objective(s): Subtract with up to 4-digit numbers using formal written methods of column subtraction

Key vocabulary: equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is..., difference, count on, strategy, partition, tens units, exchange, digit, value, hundreds, inverse, negative, positive

Key Skills for subtraction at Year 4:

- Mentally subtract any pair of two digit numbers.
- Subtract 3 digit numbers from 3 digit numbers using counting on, e.g. 426 – 278 by jumping along a line from 278 to 426
- Use formal written methods to subtract 3 and 4 digit numbers
- Practise mental subtraction strategies, e.g. Round and adjust (37—9), using place value
- Use counting on in the context of money and also when subtracting from numbers ending in zeros e.g. 4000-372
- Count backwards through zero, using negative numbers

Year 5 – Objective(s): Subtract with at least 4-digit numbers including money measures and decimals

Key vocabulary: equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is..., difference, count on, strategy, partition, tens units, exchange, digit, value, hundreds, inverse, tenths, hundredths, decimal point, decimal, negative, positive

Key Skills for subtraction at Year 5:

- Count backwards through zero, using negative numbers
- Subtract 0.1 or 0.01 to/from any decimal number with confidence, e.g. $4.83 - 0.1$
- Children need to utilise and consider a range of subtraction strategies, jottings and written methods before choosing how to calculate
- Subtract larger numbers using column subtraction or by counting up
- Begin to subtract decimal numbers using counting up: $6.2 - 3.5$
- Decide which mental methods to use and explain why

Year 6 – Objective (s): Subtract increasingly large and more complex numbers and decimal values, including money, measure and decimals with different numbers of decimal places

Key vocabulary: equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is..., difference, count on, strategy, partition, tens units, take and make, exchange, digit, value, hundreds, inverse, tenths, hundredths, decimal point, decimal, negative, positive

Key Skills for subtraction at Year 6:

- Subtract mentally with confidence – where the numbers are less than 100 or the calculation relies upon simple subtraction and place value. Examples include: $6,723 - 400$, $72 - 46$, $100 - 64$
- Subtract large numbers using formal written methods of column subtraction or counting up, e.g. $1323 - 758$
- Subtract decimal numbers using counting up
- Use negative numbers in context and calculate intervals across zero
- Children need to utilise and consider a range of mental subtraction strategies, jottings and written methods before deciding how to calculate
- Decide which methods to use and explain why

WRITTEN METHODS OF MULTIPLICATION

Stage 1 – ‘Grid method’ multiplication (2 digit x 1 digit numbers)

N.B. This method will be used as an interim method to scaffold children’s understanding between informal and formal written methods of multiplication.

$$57 \times 3 = 171$$
$$\begin{array}{r} X \quad 50 \quad 7 \\ 3 \quad \boxed{150} \quad \boxed{21} \end{array} = 171$$

Teaching points:

Children should be encouraged to estimate their answer
Children should be encouraged to check each stage of their calculations carefully

Stage 2 – Expanded vertical format (2 digit x 1 digit numbers)

$$57 \times 3 = 171$$
$$\begin{array}{r} \text{T O} \\ 57 \\ X \quad 3 \\ \hline 21 \\ 150 \\ \hline 171 \end{array}$$

Teaching points:

Children should set their calculations out in columns and label the columns
The calculation should be carried out in the following order: O×O, TxO
All digits should be written in the correct columns

Children will be introduced to the idea of using zero as a 'place holder' in the second row of the calculation (i.e. $50 \times 3 = 150$ but it is easier to calculate $5 \times 3 \times 10$)

Stage 3 – Compact vertical format (2 digit x 1 digit numbers)

$$57 \times 3 = 171$$

$$\begin{array}{r} 57 \\ X \quad 3 \\ \hline 171 \\ \hline \end{array}$$

Teaching points:

Columns should not be labelled

The calculations should be carried out in the following order: OxO, TxO.

If a number has been carried, children should strike through it once it has been added to the subsequent step.

Stage 4 – 'Grid method' multiplication (2 digit x 2 digit numbers)

N.B. This method will be used as an interim method to scaffold children's understanding between informal and formal written methods of multiplication.

$$55 \times 21 = 1155$$

$$\begin{array}{r} X \quad 50 \quad 5 \\ 20 \quad \boxed{1000} \quad \boxed{100} = 1100 \\ 1 \quad \boxed{50} \quad \boxed{5} = + \quad \underline{55} \\ \hline \quad \quad \quad \quad \quad \quad \quad \quad \underline{1155} \end{array}$$

Stage 5 – Expanded vertical format (2 digit x 2 digit numbers)

$$55 \times 21 = 1155$$

	Th	H	T	O
		5	5	
X	2	1		
<hr/>				
		5		
		5	0	
	1	0	0	
1	0	0	0	
<hr/>				
1	1	5	5	
<hr/>				

Teaching points:

Children should set their calculations out in columns and label the columns

Calculations should be done in the following order: OxO, TxO, OxT, TxT

At each stage of the calculation, the use of zero as a 'place holder' in each row will be carefully explained and modelled

All digits should be written in the correct columns

Stage 6 – Compact vertical format - 'Long multiplication' (2 digit x 2 digit numbers)

$$55 \times 21 = 1155$$

	5	5	
X	2	1	
<hr/>			
	5	5	
1	1	0	0
<hr/>			
	1		
1	1	5	5
<hr/>			

Teaching points:

Columns should not be labelled

The calculations should be carried out in the following order: OxO, TxO, OxT, TxT

If a number has been carried, children should strike through it once it has been added to the subsequent step.

Progression within Multiplication:

Year 3 – Objective(s): multiply 2-digit numbers by a single digit number

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, sets of, equal groups, times as big as, once, twice, three times..., partition, grid method, multiple, product, tens, units, value

Key Skills for multiplication at Year 3:

- Understand that multiplication is commutative, e.g. 4×8 is the same as 8×4 .
- Have instant recall of the 2x, 3x, 5x and 10x times tables in a random order. All tables need to be learned to 12th multiple
- Multiply any 2-digit number by 10 or a single-digit number by 100
- Understand the effect of multiplying whole numbers by 10 and 100
- Multiply a 1 digit number by a 2 digit number starting to use the grid
- Solve multiplication problems involving missing numbers

Year 4 – Objective(s): Multiply 2 and 3 digits by a single digit; use all multiplication tables up to 12x12

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, sets of, equal groups, times as big as, once, twice, three times..., partition, grid method, multiple, product, tens, units, value, inverse

Key Skills for multiplication at Year 4:

- Multiply 1 and 2 digit numbers by 10, 100 and 1000; to understand place value in decimal numbers with one place.
- Have instant recall of the 2x, 3x, 4x, 5x, 9x, 10x times tables up to 12th multiple in a random order; include multiplying by 0 (e.g. $5 \times 0 = 0$, $7 \times 0 = 0$) or by 1 (e.g. $5 \times 1 = 5$, $\frac{1}{2} \times 1 = \frac{1}{2}$).
- Multiply 1- digit numbers by 2-digit or friendly 3-digit numbers using grid method.
- Find doubles to double 100 and beyond, using partitioning
- Begin to double amounts of money
- Use doubling as strategy for multiplying by 2, 4, 8
- Count in multiples of 6, 7, 9, 25 and 1000

Year 5 – Objective(s): Multiply up to 4 digits by 1 or 2 digits using formal written methods

Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, sets of, equal groups, times as big as, once, twice, three times...,

partition, grid method, multiple, product, tens, units, value, inverse, square, factor, integer, decimal, short/long multiplication, 'carry'

Key Skills for multiplication at Year 5:

- Know and recite all times tables including division facts.
- Multiply 2- and 3-digit numbers by numbers ≤ 12 using grid method; multiply 2-digit by 2-digit numbers using grid method.
- Identify multiples and factors, using knowledge of multiplication tables up to 12×12
- Scale up or down by a factor of 2, 5 or 10
- Multiply integers and decimals by 10, 100, 1000
- Recognise and use squared and cubed numbers and their notations

Year 6 – Objective(s): Use formal written methods to multiply decimals with up to 2 decimal places by a single digit.

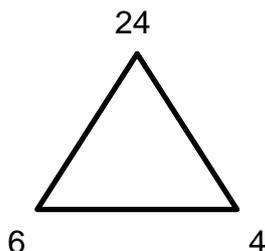
Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, sets of, equal groups, times as big as, once, twice, three times..., partition, grid method, multiple, product, tens, units, value, inverse, square, factor, integer, decimal, short/long multiplication, 'carry', tenths, hundredths, decimal

Key Skills for multiplication at Year 6:

- Recall multiplication facts up to 12×12
- Use short multiplication to multiply a 1-digit number by a number with up to 4 digits
- Use long multiplication to multiply a 2-digit by a number with up to 4 digits
- Use short multiplication to multiply a 1-digit number by a number with one or two decimal places, including amounts of money.
- Multiply fractions and mixed numbers by whole numbers.
- Multiply fractions by proper fractions.
- Use percentages for comparison and calculate simple percentages.
- Estimate answers using rounding and approximation

WRITTEN METHODS OF DIVISION

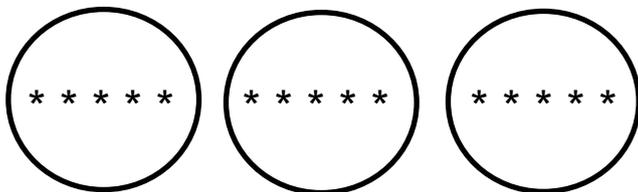
Stage 1 – Understanding the inverse relationship between multiplication and division



$$\begin{aligned}6 \times 4 &= 24 \\4 \times 6 &= 24 \\24 \div 6 &= 4 \\24 \div 4 &= 6\end{aligned}$$

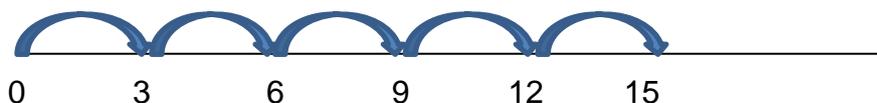
Stage 2 – Jottings to show division as sharing

$$15 \div 3 = 5$$



Stage 3 – Repeated addition using a number line (division as grouping)

$$15 \div 3 = 5$$



Teaching points:

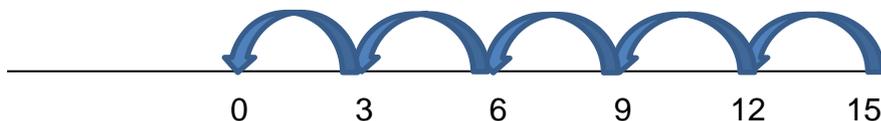
Vocabulary related to division should be introduced. The 'dividend' is the number being divided (15), the 'divisor' is the number it's being divided by (3), the 'quotient' is the answer.

Children should begin counting from zero in multiples of the divisor until they reach the dividend (or the multiple before the dividend, if there is a remainder).

Remainders should be presented as an integer, e.g. $18 \div 5 = 3 \text{ r.}3$

Stage 3 – Repeated subtraction using a number line (division as grouping)

$$15 \div 3 = 5$$



Teaching points:

Children should begin counting backwards from the dividend in multiples of the divisor until they reach zero (or the multiple before zero, if there is a remainder).

Remainders should be presented as an integer, e.g. $18 \div 5 = 3 \text{ r.}3$

Stage 4 – Chunking

N.B. This method will be used as an interim method to scaffold children's understanding between informal and formal written methods of division.

First step:

$$192 \div 6 = 32$$

$$\begin{array}{r} 32 \\ 6 \overline{) 192} \\ \underline{- 60} \quad (10 \text{ lots of } 6) \\ 132 \\ \underline{- 60} \quad (10 \text{ lots of } 6) \\ 72 \\ \underline{- 60} \quad (10 \text{ lots of } 6) \\ 12 \\ \underline{- 12} \quad (2 \text{ lots of } 6) \\ 0 \end{array}$$

Moving on to:

$$192 \div 6 = 32$$

$$\begin{array}{r} 32 \\ 6 \overline{) 192} \\ \underline{- 180} \quad (30 \text{ lots of } 6) \\ 12 \\ \underline{- 12} \quad (2 \text{ lots of } 6) \\ 0 \end{array}$$

Any remainders should be shown as integers, e.g. $28 \div 5 = 5 \text{ r.}3$. However, if the calculation is in response to a word problem, children should decide whether to round the remainder up or down, depending on the context of the question. An example of rounding down would be: 'I have 28p. Sweets are 5p each. How many can I buy? Answer: 5 (the remaining 3p is not enough to buy another sweet). An example of rounding up would be: Apples are packed into boxes of 5. There are 28 apples. How many boxes are needed to pack all of the apples? Answer: 6 (the remaining 3 apples still need to be placed into a box but it will not be full).

Stage 4 – Short division

$$192 \div 6 = 32$$

$$\begin{array}{r} 032 \\ 6 \overline{)1912} \end{array}$$

Teaching points:

A zero should be placed above the 'bus stop' where it is not possible to divide the digit by the divisor.

Extend to decimals with up to 2 decimal places

Any remainders should be presented in the context of the question, e.g. as an integer, or rounded up or down, if appropriate.

Stage 5 – Long division

$$192 \div 6 = 32$$

$$\begin{array}{r} 032 \\ 6 \overline{)192} \\ \underline{18} \\ 12 \\ \underline{12} \\ 0 \end{array}$$

Teaching points:

If appropriate to the context, any remainders should be shown as fractions, e.g. $32 \div 10 = 3 \frac{2}{10}$ which could then be simplified further to $3 \frac{1}{5}$ in its lowest form

Extend to decimals with up to 2 decimal places

$$12 \div 3 = 4$$

N.B. The 'dividend' is the number being divided (12). The 'divisor' is the number it's being divided by (3). The 'quotient' is the answer (4).

Progression within Division:

Year 3 – Objective(s): Divide 2-digit numbers by a single digit

Key vocabulary: share, share equally, one each, two each..., group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple

Key Skills for division at Year 3:

- Recall and use division facts for the 2, 3, 4, 5, 8 and 10 multiplication tables
- Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers divided by one digit
- Solve problems, in contexts, and including missing number problems, involving division.
- Pupils develop efficient mental methods, for example, using division facts (e.g. using $3 \times 2 = 6$, $6 \div 3 = 2$ and $2 = 6 \div 3$) to derive related facts ($30 \times 2 = 60$, so $60 \div 3 = 20$ and $20 = 60 \div 3$).
- Pupils develop reliable written methods for division, starting with calculations of 2- digit numbers by 1-digit numbers using an empty number line.
- Halve even numbers up to 50 and multiples of ten to 100
- Perform divisions within the tables including those with remainders, e.g. $38 \div 5$.

Year 4 – Objective(s): Divide up to 3-digit numbers by a single digit

Key vocabulary: share, share equally, one each, two each..., group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisible by, factor, dividend, divisor, quotient

Key Skills for division at Year 4:

- Use a written method to divide a 2-digit or a 3-digit number by a single-digit number.
- Give remainders as whole numbers.
- Recall multiplication and division facts for all numbers up to 12×12 .
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying and dividing by 10 and 100 and 1.
- Pupils practise to become fluent in the formal written method of short division with exact answers when dividing by a one-digit number
- Pupils practise mental methods and extend this to three-digit numbers to derive facts, for example $200 \times 3 = 600$ so $600 \div 3 = 200$
- Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as three cakes shared equally between 10 children.

Year 5 – Objective(s): Divide up to 4 digits by a single digit < or = to 12, including answers with remainders.

Key Vocabulary: share, share equally, one each, two each..., group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisible by, dividend, divisor, quotient factor quotient, prime number, prime factors, composite number (non-prime)

Key Skills for division at Year 5:

- Recall multiplication and division facts for all numbers up to 12 x 12 (as in Y4).
- Multiply and divide numbers mentally, drawing upon known facts.
- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two number.
- Solve problems involving multiplication and division where larger numbers are decomposed into their factors.
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.
- Use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
- Work out whether a number up to 100 is prime, and recall prime numbers to 19
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- Use multiplication and division as inverses. Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (e.g. $98 \div 4 = 24 \text{ r } 2 = 24\frac{1}{2} = 24.5 \approx 25$).

Year 6 – Objective(s): Divide at least 4-digit numbers by single and 2-digit numbers (including decimals)

Key Vocabulary: share, share equally, one each, two each..., group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisible by, dividend, divisor, quotient factor quotient, prime number, prime factors, composite number (non-prime), common factor

Key Skills for division at Year 6:

- Recall and use multiplication and division facts for all numbers to 12 x 12 for more complex calculations
- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context.
- Use short division where appropriate.
- Perform mental calculations, including with mixed operations and large numbers.
- Identify common factors, common multiples and prime numbers.
- Solve problems involving all 4 operations.
- Use estimation to check answers to calculations and determine accuracy, in the context of a problem.
- Use written division methods in cases where the answer has up to two decimal places.
- Solve problems which require answers to be rounded to specified degrees of accuracy.

