# SUNNYMEDE JUNIOR SCHOOL

*Learning for a Better Tomorrow*

# WRITTEN CALCULATIONS POLICY

**Ratified by Governing Body:** June 2015

**Review Frequency:** Every three years

**This Review:** June 2019

**Next Review Date:** June 2022

| Headteacher's signature | Chair of Governor's signature |
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## Rationale and Aims

This policy outlines a model progression through **written** strategies for addition, subtraction, multiplication and division in line with the 2014 National Curriculum. As children move at the pace appropriate to them, teachers will be presenting strategies and equipment appropriate to children's level of understanding. However, it is expected that the majority of children in each class will be working at age-appropriate levels as set out in the National Curriculum 2014 and in line with school policy.

This policy aims to support all staff in their understanding of teaching written methods for the four rules of addition, subtraction, multiplication and division. It contains the key pencil and paper procedures that will be taught at our school. It aims to ensure consistency, continuity and progression throughout the school and reflects a whole school agreement. However, it does not set out a system of progression that every child must work through in a set order, as strategies taught to individuals and groups of children will reflect the School's commitment to a personalised curriculum, with methods differentiated accordingly.

Pupils will progress from pictorial/ mental methods to written methods, once their depth of understanding is secure in both fluency and reasoning situations. Once one operation is secure, pupils may need to revisit more informal methods to support their understanding of other operations of mathematical concepts.

Whilst this policy focuses on written calculations in mathematics, we recognise the importance of the mental strategies and known facts that form the basis of all calculations. The following checklists outline the key skills and number facts that children are expected to develop throughout the school.

In every written method there is an element of mental processing. Sharing written methods with the teacher encourages children to think about the mental strategies that underpin them and to develop new ideas. Therefore, written recording both help children to clarify their thinking and supports and extends the development of more fluent and sophisticated mental strategies. Children will be encouraged to see mathematics as both a written and spoken language. Teachers will support and guide children through the following important stages:

- ➢ Using concrete resources to support and develop the links that are created with mathematical processes, understanding and language.
- ➢ Developing the use of pictorial representations and a mixture of words and symbols to represent numerical activities;
- ➢ Using standard symbols and conventions;
- ➢ Use of jottings to aid a mental strategy;
- ➢ Use of pencil and paper procedures;
- ➢ Use of a calculator as a checking device (although this is not a recognised skill within the national curriculum, it is still an important skill that pupils need).

This policy concentrates on the introduction of standard symbols, the use of the empty number line as a jotting to aid mental calculation and on the introduction and embedding of formal pencil and paper procedures. It is important that children do not abandon jottings and mental methods once pencil and paper procedures are introduced. Therefore, children will always be encouraged to look at a calculation/problem and then decide the best method to choose –mental calculation with or without jottings or a structured formal recording. Our long-term aim is for children to be able to select an efficient method of their choice that is appropriate for a given task.

It is expected that:

- *mental calculation methods will be practised regularly alongside their learning of written calculation methods*
- addition/subtraction and multiplication/division methods will be taught alongside each other so that pupils can see and use the relationship between them
- pupils will be taught to recognise and use a range of vocabulary relating to each of the four rules
- Concrete resources, models and images will be used to support learning of methods at all ages and stages, if required, though particularly for pupils with special education needs
- pupils will be taught to estimate their answers first and to check calculations with a variety of strategies, including the inverse operation.
- pupils will have opportunities to use and apply mental and written strategies in a range of problem solving, investigative and 'real-life' contexts
- children of all abilities will be able to show mastery of maths at their level of understanding.

## PROGRESSION IN WRITTEN STRATEGIES FOR ADDITION AND SUBTRACTION
To add and subtract successfully, by the end of each year children should be able to:

Year 3:
- add and subtract numbers mentally, including:
- a three-digit number and ones
- a three-digit number and tens
- a three-digit number and hundreds
- add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

Year 4:
- add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.

Year 5:
- add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
- add and subtract numbers mentally with increasingly large numbers
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

Year 6:

- perform mental calculations, including with mixed operations and large numbers
- identify common factors, common multiples and prime numbers
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why Mathematics
- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.


## PROGRESSION IN WRITTEN STRATEGIES FOR MULTIPLICATION AND DIVISION
To multiply and divide successfully, by the end of each year children should be able to:


Year 3:
- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

Year 4:
- recall multiplication and division facts for multiplication tables up to 12 × 12
- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout #solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

Year 5:
- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- know and use the vocabulary of prime numbers, prime factors and composite (nonprime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- multiply and divide numbers mentally drawing upon known facts
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000 Mathematics – key stages 1 and 2 33 Statutory requirements
- recognise and use square numbers and cube numbers, and the notation for squared ( 2 ) and cubed (3 )
- solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes

- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

Year 6:
- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- solve problems involving addition, subtraction, multiplication and division

# WRITTEN METHODS OF ADDITION

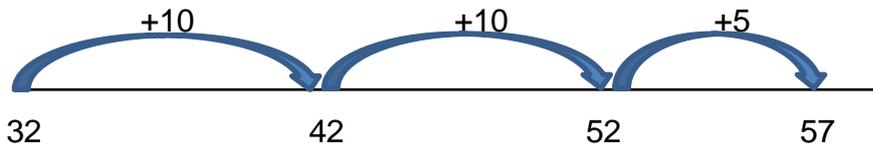**Upon entry to Year 3, children will use a variety of informal jottings**

Method 1

50

$32 + 25$      $50 + 7 = 57$

7

Method 2

$32 + 25$

$30 + 2 + 20 + 5 = 57$

Method 3

+10          +10          +5

32          42          52          57

Teaching point:

The Year 3 teachers will ascertain which method(s) the children in their group have been taught in KS1. Using this knowledge, they will support and scaffold progression onto more formal methods of recording (Stage 1, see below) using the method(s) that the children currently use.

**Stage 1 – Partitioning using 2 digit numbers**

Without carrying (e.g. 32+25):

T   $30 + 20 = 50$
O   $2 + 5 = 7$

$50 + 7 = 57$

With carrying (e.g. 35+27):

T   $30 + 20 = 50$
O   $5 + 7 = 12$

$50 + 12 = 62$

**Stage 2 – Expanded column addition using 2 digit numbers**

Without carrying (e.g. 32+25):

```
    T    O
   30 +  2
 + 20 +  5
 _____
   50 +  7
 _____
```

With carrying (e.g. 35+27):

```
    T    O
   30 +  5
 + 20 +  7
 _____
   50 + 12
 _____
```

Moving on to:

```
    T    O
   30 +  5
 + 20 +  7
 _____
   60 +  2
 _____
      1̶0̶
```

Teaching points:

Children should be encouraged to lay their calculations out in columns and to label the columns
Children should always add the digits in the 'ones' column first
Children should strike through the number that has been carried once they have added it to the column total
Extend to three digit numbers

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**Stage 3 – Compact column method (Children will be expected to use this method to add whole numbers together by the end of Year 4)**

First step (with column labels):

```
   T   O
   3   5
 + 2   7
 _____
   6   2
 _____
      1̶
```

Second step (without column labels):

```
    35
   +27
   _____
    62
   _____
     1̶
```
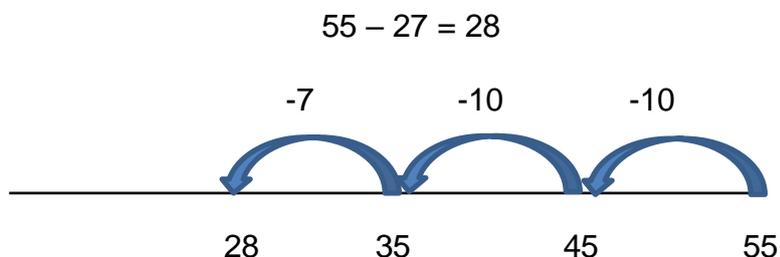
Teaching points:

Extend to three digit numbers.  This concept will be taught in gradual stages.  In the first instance, the calculations will require carrying from the 'ones' to the 'tens' column, then the 'tens' to the 'hundreds' column.  Finally, carrying will be required from the 'ones' to the 'tens' and the 'tens' to the 'hundreds' within the same calculation
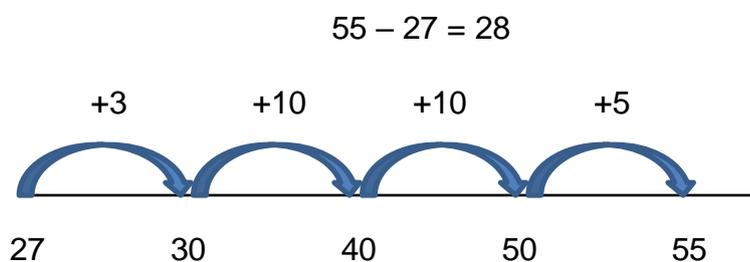Extend to decimal numbers.

# WRITTEN METHODS OF SUBTRACTION

## Stage 1 - Subtraction using an empty number line

Method 1 (counting backwards):

$$55 - 27 = 28$$

-7          -10          -10

28        35          45          55

Method 2 (counting forwards):

$$55 - 27 = 28$$

+3          +10          +10          +5

27        30        40        50        55

Teaching points:
Using method 2, children should be encouraged to firstly jump to the next multiple of 10.
Confusion may occur as the children are counting on from the smallest to the largest number.

Method 3 (counting forward using a more efficient method):

$$55 - 27 = 28$$

+3          +20          +5

27        30                50        55
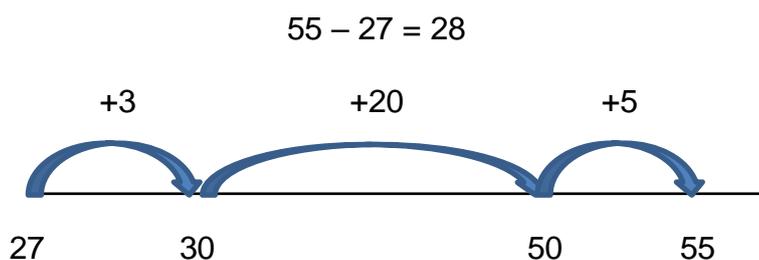
Method 4 (counting backwards using a more efficient method):

$$55 - 27 = 28$$

-3          -20          -5

27        30                50        55
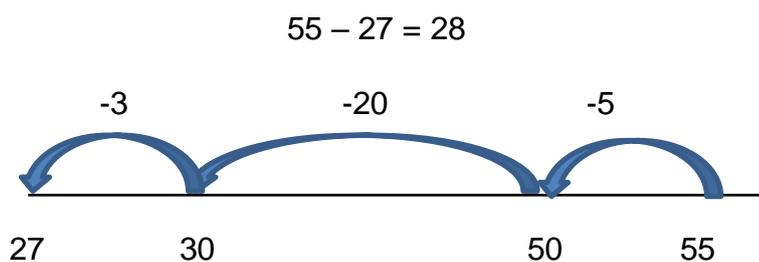
Teaching points:
An empty number line can also be used at any stage of development for the following areas:
- Decimals e.g. 4.2 – 3.68
- Elapse time e.g. Find the difference between 3.55pm and 6.20pm
- Numbers containing zeros e.g. 6003 – 1987
- Conversion in measure (capacity/length etc.) e.g. 5 litres – 860ml

## Stage 2 – Expanded column subtraction using 2 digit numbers

Without exchanging (e.g. 65-22):       With exchanging (e.g. 61-27):       Moving on to:

|  | T | O |
|---|---|---|
|  | 60 | 5 |
| - | 20 | 2 |
|  | 40 + | 3 |

|  |  | T | O |
|---|---|---|---|
|  | 50 | 60 | 10+1 |
| - |  | 20 | 7 |
|  |  | 30 + | 4 |

|  |  | T | O |
|---|---|---|---|
|  | 50 | 60 | 11 |
| - |  | 20 | 7 |
|  |  | 30 + | 4 |

Teaching points:
Children should be encouraged to lay their calculations out in columns and to label the columns
Children should always subtract the digits in the 'ones' column first
Extend to 3 digit numbers

## Stage 3 – Compact column method (decomposition) (Children will be expected to use this method to subtract whole numbers together by the end of Year 4)

Step 1 (2 digit numbers without exchanging):

```
   T O
   8 7
 - 5 3
   3 4
 ─────
```

Step 2 (3 digit numbers without exchanging):

```
    257
  - 123
    134
  ─────
```

Step 3 (2 digit numbers with exchanging):

```
   T O
  8  1
   9 1
 - 3 6
   5 5
 ─────
```

Step 4 (3 digit numbers with exchanging):

```
   1 12 1
    2 3 5
  - 1 7 6
      5 9
    ─────
```

# WRITTEN METHODS OF MULTIPLICATION

## Stage 1 – 'Grid method' multiplication (2 digit x 1 digit numbers)

*N.B. This method will be used as an interim method to scaffold children's understanding between informal and formal written methods of multiplication.*

57 x 3 = 171

| X | 50 | 7 | |
|---|-----|-----|-------|
| 3 | 150 | 21 | = 171 |

Teaching points:

Children should be encouraged to estimate their answer
Children should be encouraged to check each stage of their calculations carefully

## Stage 2 – Expanded vertical format (2 digit x 1 digit numbers)

57 x 3 = 171

```
    T O
    5 7
  X   3
    2 1
  1 5 0
  1 7 1
```

Teaching points:
Children should set their calculations out in columns and label the columns
The calculation should be carried out in the following order:  OxO, TxO
All digits should be written in the correct columns
Children will be introduced to the idea of using zero as a 'place holder' in the second row of the calculation (i.e. 50x3 = 150 but it is easier to calculate 5x3x10)

**Stage 3 – Compact vertical format (2 digit x 1 digit numbers)**

57 x 3 = 171

```
    5 7
  X   3
  1 7 1
 ―――――――
    2
```

Teaching points:
Columns should not be labelled
The calculations should be carried out in the following order: OxO, TxO.
If a number has been carried, children should strike through it once it has been added to the subsequent step.

**Stage 4 – 'Grid method' multiplication (2 digit x 2 digit numbers)**

*N.B. This method will be used as an interim method to scaffold children's understanding between informal and formal written methods of multiplication.*

55 x 21 = 1155

| X | 50 | 5 | | |
|---|------|-----|-----|------|
| 20 | 1000 | 100 | = | 1100 |
| 1 | 50 | 5 | = + | 55 |
| | | | | 1155 |

**Stage 5 – Expanded vertical format (2 digit x 2 digit numbers)**

55 x 21 = 1155

```
Th H T  O
       5 5
   X   2 1
         5
       5 0
     1 0 0
   1 0 0 0
   _____
   1 1 5 5
   _____
```

Teaching points:
Children should set their calculations out in columns and label the columns
Calculations should be done in the following order: OxO, TxO, OxT, TxT
At each stage of the calculation, the use of zero as a 'place holder' in each row will be carefully explained and modelled
All digits should be written in the correct columns

**Stage 6 – Compact vertical format  - 'Long multiplication' (2 digit x 2 digit numbers)**

55 x 21 = 1155

```
       5 5
   X   2 1
   _____
       5 5
   1 1 0 0
      ⁄1
   _____
   1 1 5 5
   _____
```
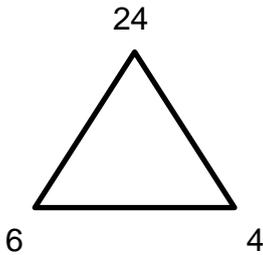
Teaching points:
Columns should not be labelled
The calculations should be carried out in the following order: OxO, TxO, OxT, TxT
If a number has been carried, children should strike through it once it has been added to the subsequent step.
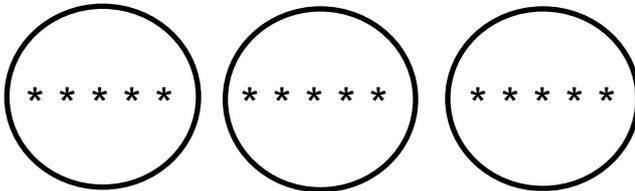
# WRITTEN METHODS OF DIVISION

## Stage 1 – Understanding the inverse relationship between multiplication and division

24

6          4

6 x 4 = 24
4 x 6 = 24
24 ÷ 6 = 4
24 ÷ 4 = 6

## Stage 2 – Jottings to show division as sharing

15 ÷ 3 = 5

* * * * *        * * * * *        * * * * *

## Stage 3 – Repeated addition using a number line (division as grouping)

15 ÷ 3 = 5

0      3      6      9      12      15
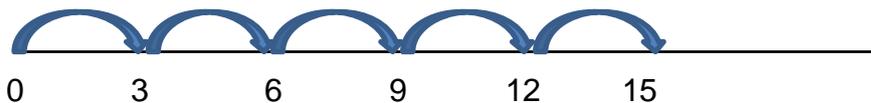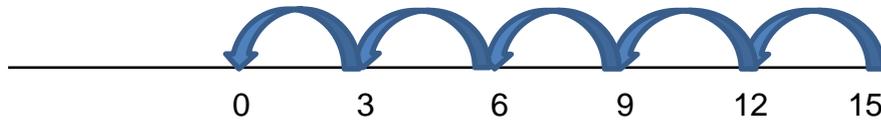
Teaching points:
Vocabulary related to division should be introduced.  The 'dividend' is the number being divided (15), the 'divisor' is the number it's being divided by (3), the 'quotient' is the answer.
Children should begin counting from zero in multiples of the divisor until they reach the dividend (or the multiple before the dividend, if there is a remainder).

Remainders should be presented as an integer, e.g. 18÷5 = 3 r.3

## Stage 3 – Repeated subtraction using a number line (division as grouping)

15 ÷ 3 = 5



0     3     6     9     12     15

Teaching points:
Children should begin counting backwards from the dividend in multiples of the divisor until they reach zero (or the multiple before zero, if there is a remainder).
Remainders should be presented as an integer, e.g. 18÷5 = 3 r.3

## Stage 4 – Chunking
*N.B. This method will be used as an interim method to scaffold children's understanding between informal and formal written methods of division.*

First step:                                                    Moving on to:

192 ÷ 6 = 32                                              192 ÷ 6 = 32

```
      32                                                        32
    _____                                                    _____
6 | 192                                                   6 | 192
   - 60    (10 lots of 6)                                    - 180   (30 lots of 6)
   ----                                                        ----
    132                                                         12
   - 60    (10 lots of 6)                                    -  12    (2 lots of 6)
   ----                                                        ----
     72                                                          0
    -60    (10 lots of 6)
   ----
     12
   - 12    (2 lots of 6)
   ----
      0
```

Any remainders should be shown as integers, e.g. 28÷5 = 5 r.3.  However, if the calculation is in response to a word problem, children should decide whether to round the remainder up or down, depending on the context of the question.  An example of rounding down would be: 'I have 28p. Sweets are 5p each.  How many can I buy?  Answer: 5 (the remaining 3p is not enough to buy another sweet).  An example of rounding up would be:  Apples are packed into boxes of 5. There are 28 apples.  How many boxes are needed to pack all of the apples?  Answer: 6 (the remaining 3 apples still need to be placed into a box but it will not be full).

## Stage 4 – Short division

192 ÷ 6 = 32

```
      0 3 2
    ┌─────────
  6 │ 1̸ ₁9 ₁2
```

Teaching points:

A zero should be placed above the 'bus stop' where it is not possible to divide the digit by the divisor.
Extend to decimals with up to 2 decimal places
Any remainders should be presented in the context of the question, e.g. as an integer, or rounded up or down, if appropriate.

## Stage 5 – Long division

192 ÷ 6 = 32

```
      0 3 2
    ┌─────────
  6 │ 1 9 2
    │ 1 8
    └─────────
        1 2
        1 2
    ─────────
          0
```

Teaching points:

If appropriate to the context, any remainders should be shown as fractions, e.g. 32 ÷ 10 = 3 2/10 which could then be simplified further to 3 1/5 in its lowest form
Extend to decimals with up to 2 decimal places

**12÷3 = 4**

**N.B. The 'dividend' is the number being divided (12). The 'divisor' is the number it's being divided by (3). The 'quotient' is the answer (4).**

**PROGRESSION IN KEY VOCABULARY**

|  | **Addition** | **Subtraction** | **Multiplication** | **Division** |
|---|---|---|---|---|
| **Y3** | add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, units, ones, tens, hundreds, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', expanded, compact, digits, inverse | equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least, count back, how many left, how much less is…, difference, count on, strategy, partition, tens units, exchange, digit, value, hundreds | groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, sets of, equal groups, times as big as, once, twice, three times…, partition, grid method, multiple, product, tens, units, value | share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple |
| **Y4** | add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, units, ones, tens, hundreds, thousands, partition, addition, column, tens boundary, hundreds boundary, thousands boundary, increase, vertical, 'carry', expanded, compact, digits, inverse | equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is…, difference, count on, strategy, partition, tens units, exchange, digit, value, hundreds, inverse, negative, positive | groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, sets of, equal groups, times as big as, once, twice, three times…, partition, grid method, multiple, product, tens, units, value, inverse | share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisible by, factor, dividend, divisor, quotient |
| **Y5** | add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, ones, units, tens, hundreds, thousands, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', expanded, compact, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths. | equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is…, difference, count on, strategy, partition, tens units, exchange, digit, value, hundreds, inverse, tenths, hundredths, decimal point, decimal, negative, positive | groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, sets of, equal groups, times as big as, once, twice, three times…, partition, grid method, multiple, product, tens, units, value, inverse, square, factor, integer, decimal, short/long multiplication, 'carry' | share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisible by, dividend, divisor, quotient factor quotient, prime number, prime factors, composite number (non-prime) |
| **Y6** | add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, units, ones, tens, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', expanded, compact, thousands, hundreds, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths. | equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is…, difference, count on, strategy, partition, tens units, take and make, exchange, digit, value, hundreds, inverse, tenths, hundredths, decimal point, decimal, negative, positive | groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, sets of, equal groups, times as big as, once, twice, three times…, partition, grid method, multiple, product, tens, units, value, inverse, square, factor, integer, decimal, short/long multiplication, 'carry', tenths, hundredths, decimal | share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisible by, dividend, divisor, quotient factor quotient, prime number, prime factors, composite number (non-prime), common factor |